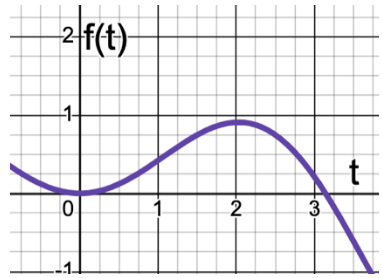
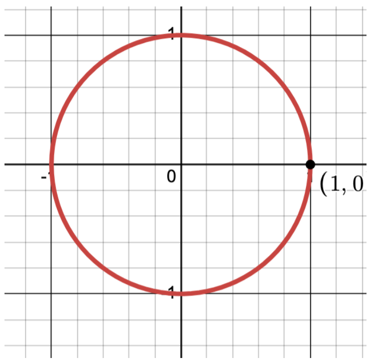
5.1 The Unit Circle: Connection between arc length and angles

Goal is to define new functions of a real number, t. These functions will be defined by the process discussed in this section



The Unit Circle.

When discussing angles in radians, we often consider the "Unit Circle". $x^2 + y^2 = 1$



Are the following points on the unit circle? $A\left(\frac{3}{5}, \frac{4}{5}\right)$ $B\left(\frac{1}{2}, \frac{-2}{3}\right)$

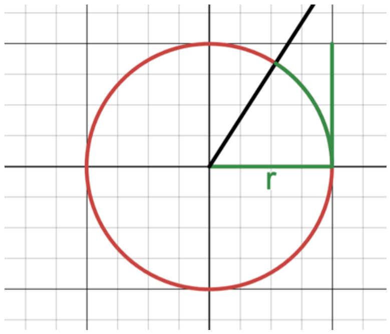
Find a point on the unit circle given the following conditions:

Ex. The point $P\left(\frac{1}{3}, y\right)$ is on the unit circle in Quadrant IV. Find y.

Radian Measure Revisited

We said 1 revolution = ________________. (arbitrarily chosen at the time)

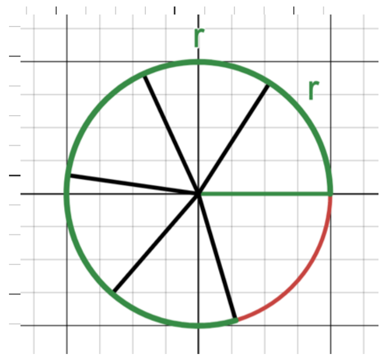
But why 2π? What is a radian?



Definition:
One radian is defined as the angle subtended from the center of a circle which intercepts an arc equal in length to the radius (see animation https://www.youtube.com/watch?v=MgbKapLpgUA)

So the measure of θ is ________________?

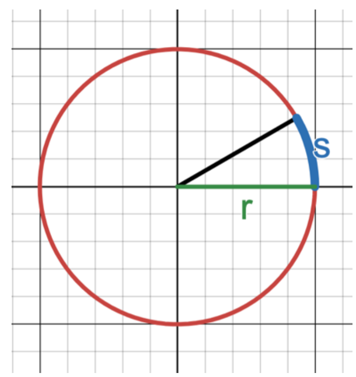
How many radians does it take to go all the way around?__________________



Arc Length

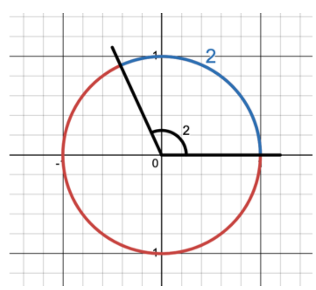
Proportions give us a relationship between the length, s, of an arc and the measure, θ, of an angle measured

in ______________



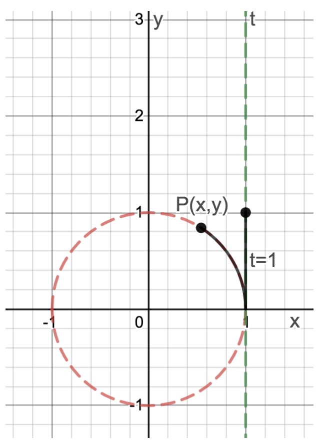
Example:

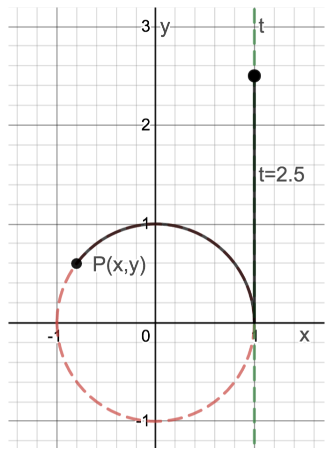
In the unit circle, this becomes ____________________

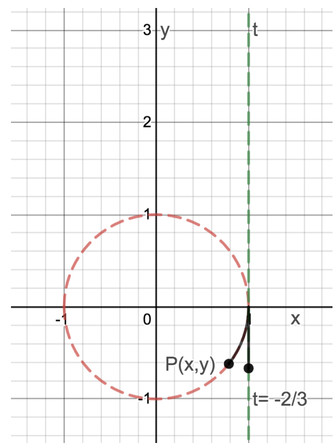


Exploring Arc Length: "Unit Circle Wrap Process"

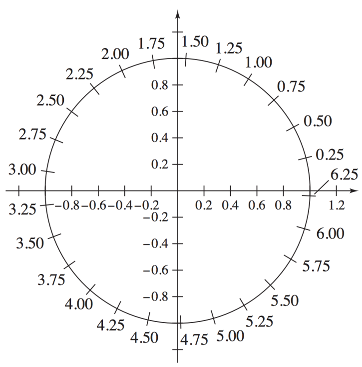
Consider the real number line corresponding values of t aligned next to the unit circle as shown where positive values of t are shown upward, negative are downward. If this number line were wrapped around the unit circle, then every number t would correspond to a point P(x,y) on the unit circle found by using $|t|$ as the arc length. (Notice: Angles are not being discussed here)







We can approximate the coordinates for a given arc length using a unit circle with units marked around the circumference,



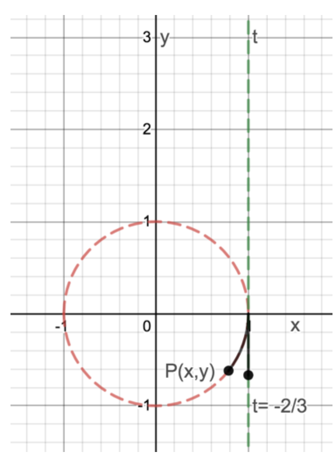
How would you (roughly) find the *location* (don't worry about coordinates yet) of the point corresponding to without using the above graph?:

a). $t = \pi / 4$?

b) $t = 2\pi / 3$?

c) $t = 9\pi / 8$

d) $t = -11\pi / 6$



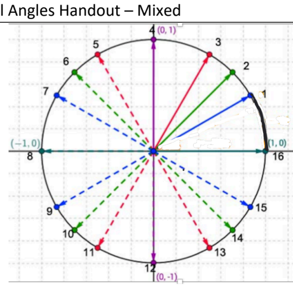
Since in the unit circle, ________________________,
given any real number, we can find the location of its terminal point on the unit circle by

____

In this context, we use the terminology "reference number" instead of "reference angle"

We have been doing this, but thinking in terms of _____________________

Angle Worksheet 3 -Special Angles Handout – Mixed



Given that all the "blue angles" have a reference angle of 30 degrees or $\pi/6$ radians, write the angle measures, both in radians AND degrees, for each of the blue angles.

1) 30° $\pi/6$

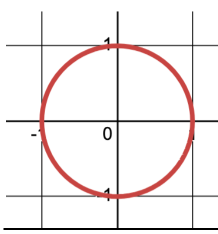
7) 150° $5\pi/6$

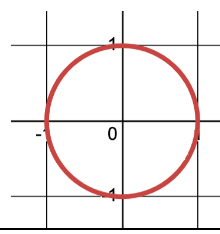
9) 210° $7\pi/6$

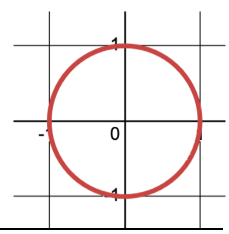
How do we find actual exact values of those points?

Find the (exact) terminal point for the real number:

$t = \pi$ $\quad$ $t = \pi / 2$ $\quad$ $t = -6\pi$

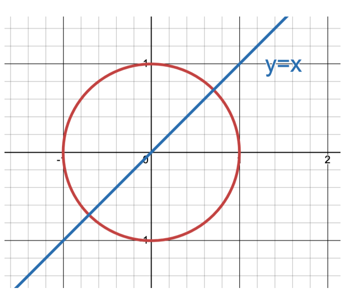




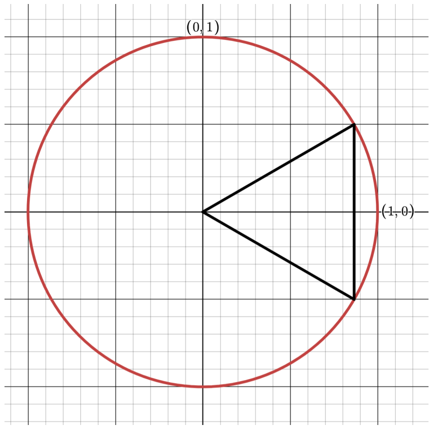


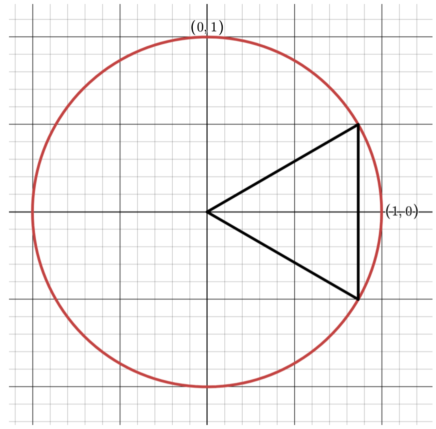
For what other real numbers (angles) can we find exact values:

$t = \pi / 4$



$t = \pi / 6$





$t = \pi / 3$ done similarly

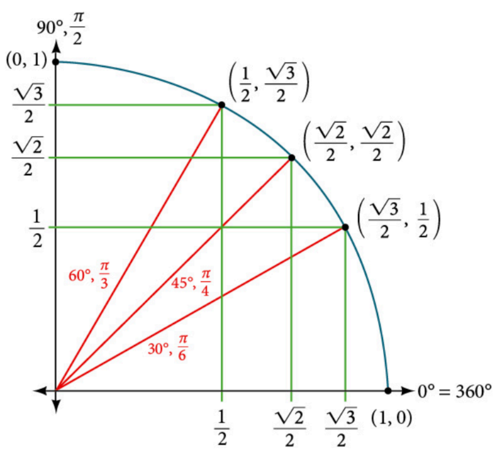
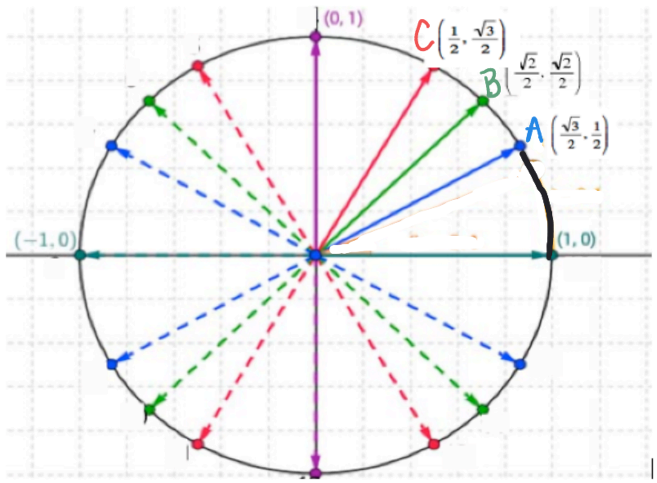


Figure 14

<u>Symmetry and Important Points on the Unit Circle.</u>

We are often interested in looking where the terminal side of some of the "key numbers t" (or angles θ) mentioned earlier intersect the "unit circle" $x^2 + y^2 = 1$. We have now found the points in the first quadrant shown.



Again, answer the following:

Coordinates of point which is symmetric to A relative to the y axis____

Coordinates of point which is symmetric to A relative to the x axis___

Coordinates of point which is symmetric to A relative to the origin.___

List the points in ascending order according to their y value. ___________

List the points in ascending order according to their x value. ___________

Example:

Find the point on the unit circle corresponding to $t = \frac{5\pi}{3}$ ___________________

Find the y value of the point on the unit circle corresponding to $t = \frac{5\pi}{6}$

Find the x value of the point on the unit circle corresponding to $t = \frac{-\pi}{4}$ ___________________

Find the coordinates of the point on the unit circle corresponding to $t = \frac{4\pi}{3}$ ___________________

(Angle Worksheet 4)

5.2 Trigonometric Functions of Real Numbers

*(You might find it helpful to review your general knowledge of functions 2.1-2.6_*

Definition of the Trigonometric Functions

Let t be any real number and let $P(x, y)$ be the terminal point on the unit circle determined by t (using the "wrap" process discussed earlier). Define:

$$\sin(t) = y \qquad \csc(t) = \frac{1}{y} \quad (y \neq 0)$$

$$\cos(t) = x \qquad \sec(t) = \frac{1}{x} \quad (x \neq 0)$$

$$\tan(t) = \frac{y}{x} \ (x \neq 0) \qquad \cot(t) = \frac{x}{y} \quad (y \neq 0)$$

Notation: can be written $\sin t$ or $\sin(t)$

Examples Quadrant 1

$\sin(\pi / 6) =$ ______________ $\csc(\pi / 6) =$ ______________

$t = \pi / 6$ $\cos(\pi / 6) =$ ______________ $\sec(\pi / 6) =$ ______________

$\tan(\pi / 6) =$ ______________ $\cot(\pi / 6) =$ ______________

$\sin(\pi / 4) =$ ______________ $\csc(\pi / 4) =$ ______________

$t = \pi / 4$ $\cos(\pi / 4) =$ ______________ $\sec(\pi / 4) =$ ______________

$\tan(\pi / 4) =$ ______________ $\cot(\pi / 4) =$ ______________

$\sin(\pi / 3) =$ ______________ $\csc(\pi / 3) =$ ______________

$t = \pi / 3$ $\cos(\pi / 3) =$ ______________ $\sec(\pi / 3) =$ ______________

$\tan(\pi / 3) =$ ______________ $\cot(\pi / 3) =$ ______________

Reciprocal Identities

$\sec(\pi/3) =$ ______________ $\csc(\pi/4) =$ ______________ $\cot(\pi/6) =$ ______________

Examples Quadrantal Numbers (Angles)

$\sin(0) =$ ______________ $\csc(0) =$ ______________

$\cos(0) =$ ______________ $\sec(0) =$ ______________

$\tan(0) =$ ______________ $\cot(0) =$ ______________

$\sin(\pi/2) =$ ______________ $\csc(\pi/2) =$ ______________

$\cos(\pi/2) =$ ______________ $\sec(\pi/2) =$ ______________

$\tan(\pi/2) =$ ______________ $\cot(\pi/2) =$ ______________

$\sin(\pi) =$ ______________ $\csc(\pi) =$ ______________

$\cos(\pi) =$ ______________ $\sec(\pi) =$ ______________

$\tan(\pi) =$ ______________ $\cot(\pi) =$ ______________

$\sin(3\pi/2) =$ ______________ $\csc(3\pi/2) =$ ______________

$\cos(3\pi/2) =$ ______________ $\sec(3\pi/2) =$ ______________

$\tan(3\pi/2) =$ ______________ $\cot(3\pi/2) =$ ______________

Domain of Trigonometric Functions

Examples: Evaluating Trig Functions in Other Quadrants

$\sin(\pi/3) =$ ______________ $\sin(2\pi/3) =$ ______________

$\sin(4\pi/3) =$ ______________ $\sin(5\pi/3) =$ ______________

Notice:

$\cos(\pi/6) =$ ______________ $\cos(5\pi/6) =$ ______________

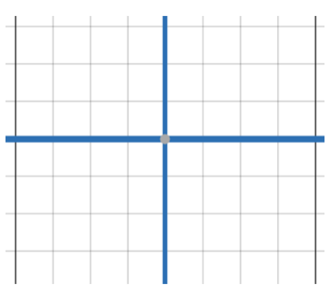
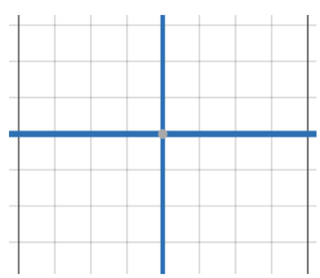
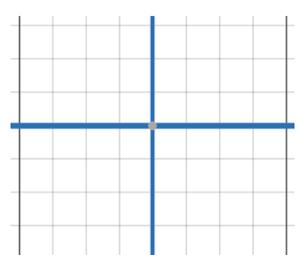
$\cos(7\pi/6) =$ ______________ $\cos(11\pi/6) =$ ______________

$\tan(\pi/4) =$ ______________ $\tan(3\pi/4) =$ ______________

$\tan(5\pi/4) =$ ______________ $\tan(7\pi/4) =$ ______________

So Trig Functions having the same reference number (angle) have the same absolute value, but may differ in sign depending on the quadrant.

Signs: x _________ y _________ y/x _____________

Example: Quadrant of a Terminal Point

In what quadrant is $\sin t < 0$ and $\sec t > 0$ _______________

Examples: Evaluating Trig Functions Using only Reference Numbers and Quadrant Signs

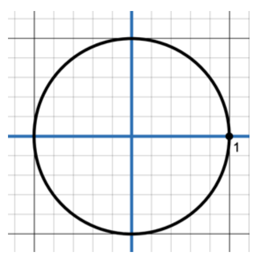
$\cos(11\pi/6) =$ ______________ $\sin(4\pi/3) =$ _______________

$\tan(5\pi/6) =$ ______________ $\sec(5\pi/4) =$ _______________

$\csc(5\pi/3) =$ ______________ $\tan(-5\pi/4) =$ _______________

Even Odd Properties of Trigonometric Functions

Recall: Even Function Odd Function



$\sin(-t) =$ ______________ $\cos(-t) =$ ______________ $\tan(-t) =$ ______________

odd even odd

Example: Using Even/Odd Properties

$\cos(-2\pi/3) =$ ______________ $\sin(-7\pi/4) =$ ______________

Estimating Trig Values With a Calculator –

$\cos(\pi/8) =$ ______________ $\sin(-5\pi/12) =$ ______________ $\tan(3) =$ ______________

$\sec(4) =$ ______________ $\cot(100) =$ ______________ $\csc(1.3) =$ ______________

Very IMPORTANT: ______________________________

Pythagorean Identities

Example: We use the notation $\cos^2(t)$ to mean _______________

Compute

$$\cos^2(\pi/3) + \sin^2(\pi/3)$$

Pythagorean Identities		
$\cos^2(t) + \sin^2(t) =$ __	$\cos^2(t) + \sin^2(t)$	$\cos^2(t) + \sin^2(t)$

Example: Finding All Trig Values Given the Value of One of Them

Given that $\cos(t) = -\dfrac{3}{5}$ and t is in Quadrant III, find the values of the other 5 trig functions as t

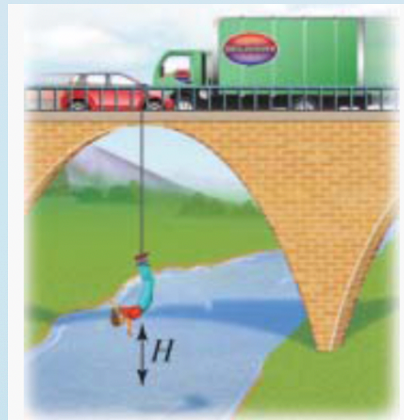
Example: Writing One Trig Function in Terms of Another

Write $\tan(t)$ in terms of $\sin(t)$ for t in Quadrant III

Applications

82. **Bungee Jumping** A bungee jumper plummets from a high bridge to the river below and then bounces back over and over again. At time t seconds after her jump, her height H (in meters) above the river is given by $H(t) = 100 + 75e^{-t/20}\cos\left(\frac{\pi}{4}t\right)$. Find her height at the times indicated in the table.

t	$H(t)$
0	
1	
2	
4	
6	
8	
12	



Explore using Desmos

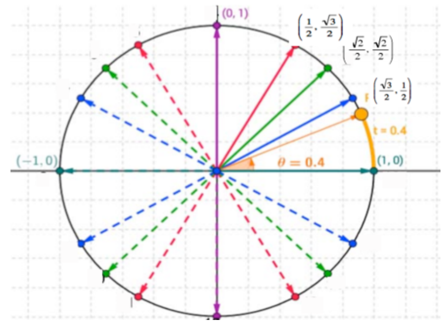
Introduction to Solving Trig Equations

(This material is covered in the book in section 7.4, but I am introducing the basics here. These basics will be practiced on a worksheet and will be on the test for this unit)

Going "backwards" from finding trig. values

$$\sin\left(\frac{\pi}{3}\right) = \_\_\_\_\_\_\_\_\_ \qquad \sin(\quad) = \frac{\sqrt{3}}{2}$$

"What is the y value of a point on the unit circle for input π/3?" "What input(s) have a y value of $\sqrt{3}/2$



Basic Equations: Solving sine and cosine equations for special number inputs

Solve: $\sin(t) = \frac{\sqrt{2}}{2}$

This is saying, find the real number (arc length or corresponding angle, in radians) whose corresponding point on the unit circle has Y value of $\frac{\sqrt{2}}{2}$

Why Y value? __________________________

How many terminal sides are there corresponding to this______________________

How many values of t? (or think in angles) __________________________

How do we express infinitely many answers?__________________________

Sometimes we are asked to solve for t on a restricted domain:

Solve: $\sin(t) = \frac{\sqrt{2}}{2}$ for $0 < t < \pi/2$ ________________________

Solve: $\sin(t) = \frac{\sqrt{2}}{2}$ for $0 \le t < 2\pi$ ________________________

Solve: $\sin(t) = \frac{\sqrt{2}}{2}$ for $0 \le t < 4\pi$ ________________________

Solve: $\sin(t) = \frac{\sqrt{2}}{2}$ for $-2\pi \le t < 0$ ________________________

Examples: While you are learning the process, I highly encourage you to draw the unit circle and find the location of the terminal sides corresponding to the solution.

Solve: $\cos(t) = -1/2$

This is asking us to find the real number (arc length or corresponding angle, in radians) whose corresponding point on the unit circle has_____________value of -1/2

Solutions: ________________________

Solve: $\cos(t) = -1/2$ for $0 < t < \pi$ ________________________

Solve: $\cos(t) = 1$

This is asking us to find the real number (arc length or corresponding angle, in radians) whose corresponding point on the unit circle has_____________value of 1

Solutions: ________________________

Solve: $\cos(t) = 1$ for $0 \le t < 2\pi$ ________________________

Solve: $\sin(t) = 0$

This is asking us to find the real number (arc length or corresponding angle, in radians) whose corresponding point on the unit circle has_____________value of 0

Solutions: ________________________

Solve: $\sin(t) = 0$ for $0 < t < \pi / 2$ ________________________

Solve: $\sin(t) = -\frac{\sqrt{2}}{2}$

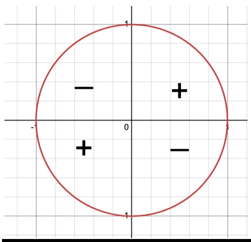
This is asking us to find the real number (arc length or corresponding angle, in radians) whose corresponding point on the unit circle has_____________value of $-\frac{\sqrt{2}}{2}$

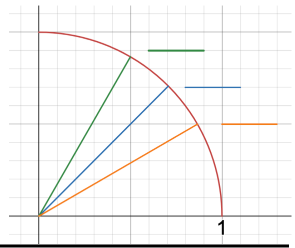
Solutions: ________________________

Solve: $\sin(t) = -\frac{\sqrt{2}}{2}$ for $-\pi / 2 < t < \pi / 2$ ________________________

Basic Equations: Solving tangent equations for special number inputs

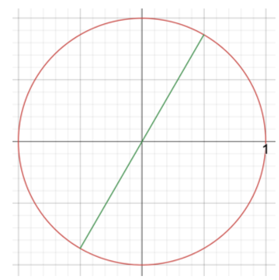
For solving equation with tangent, we need to be familiar with the tangent values in the first quadrant, and the tangent signs as discussed earlier.





Solve: $\tan(t)=\sqrt{3}$

This is asking us to find the real number (arc length or corresponding angle, in radians) whose tangent has value of $\sqrt{3}$



What is the reference angle? ______________.

What quadrants have positive tangent values? _______

Solve: $\tan(t)=\sqrt{3}$ for $0<t<2\pi$ __________________________

All Solutions: __________________________

Solve: $\tan(t)=-1$

This is asking us to find the real number (arc length or corresponding angle, in radians) whose tangent has value of -1

What is the reference angle? ______________.

What quadrants have positive tangent values? _______

Solve: $\tan(t)=-1$ for $0<t<2\pi$ __________________________

All Solutions: __________________________

(Solving Basic Equations Worksheet)

Other Trig Functions: Solve for $0<t\le 2\pi$

$\sec(t)=2$ $\qquad\qquad$ $\cot(t)=0$ $\qquad\qquad$ $\csc(t)=-2/\sqrt{3}$

Forwards and backwards, mixed: Simplify or Solve

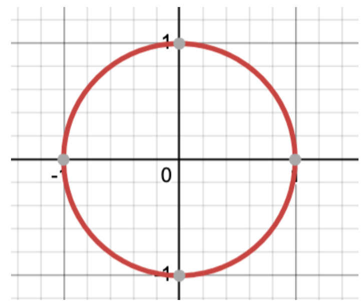
$\tan(t)=\sqrt{3}/3$ $\qquad\qquad$ $\cos(5\pi/3)$ $\qquad\qquad$ $\cot(\pi)$ $\qquad\qquad$ $\sin(t)=-\sqrt{2}/2$

5.3i Graphing the Sine and Cosine Function part i

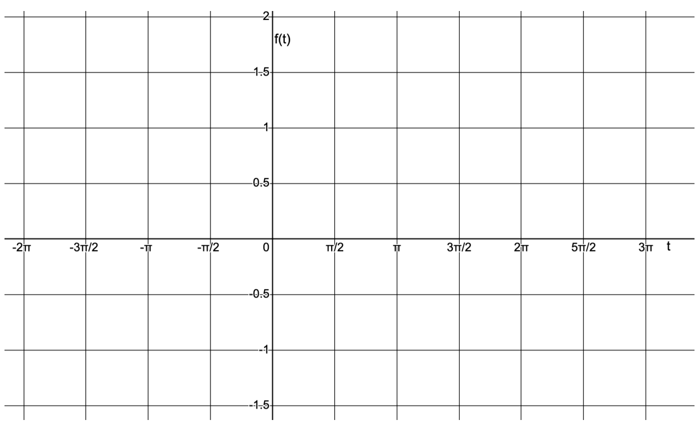
$$f(t) = \sin(t)$$

t_1	$f(t_1)$
0	
$\frac{\pi}{6}$	
$\frac{\pi}{4}$	
$\frac{\pi}{3}$	
$\frac{\pi}{2}$	

t_1	$f(t_1)$
π	
$\frac{3\pi}{2}$	
2π	



Note choice of scale on t axis.



Picture the unit circle. What happens to the ________________ values as t increases from $0 \to \pi/2$

What does y do as t goes from

$\pi/2 \to \pi$ _________________________

$\pi \to 3\pi/2$ _________________________

$3\pi/2 \to 2\pi$ _________________________

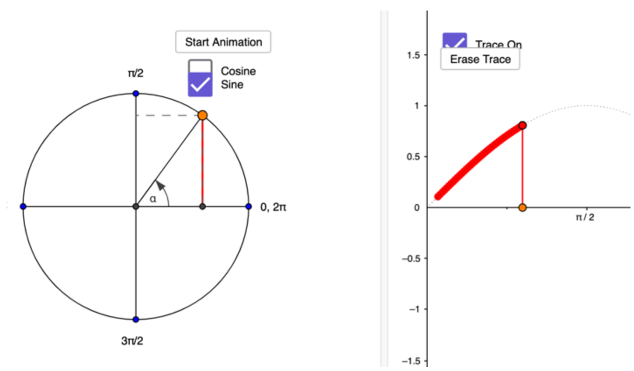
$2\pi \to 5\pi/2$ _________________________

A function is called periodic with period p if $f(t+p)=f(t)$. Since $\sin(t+2\pi)=\sin(t)$,

the function $f(t)=\sin(t)$ is ________________ with period ________________

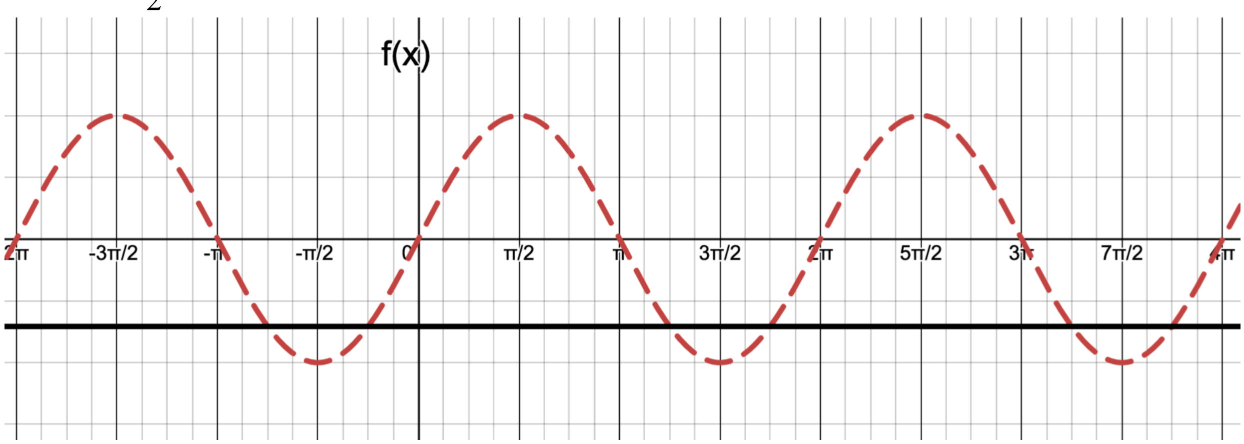
How domain, range, period, even/odd, can be seen on graph.

This type of "sinusoidal wave" can be used to measure many physical phenomena.
Annimation: See
https://www.geogebra.org/m/cNEtsbvC
Or
https://www.geogebra.org/m/G9mjcC7D



Using the graph to help visualize\ solutions to trig. Equations. As done in an earlier example. solve

$\sin(t)=-\frac{\sqrt{2}}{2}$. What would be the solution if $-\pi/2<t<\pi/2$



$f(t) = \cos(t)$

The graph of $f(t) = \cos(t)$ can be generated similarly. In particular, plot the points corresponding to the quadrantal inputs (angles),

t_1	$f(t_1)$
0	
$\frac{\pi}{2}$	
π	
$\frac{3\pi}{2}$	
2π	

then using the unit circle, consider what ___________ does as t goes from

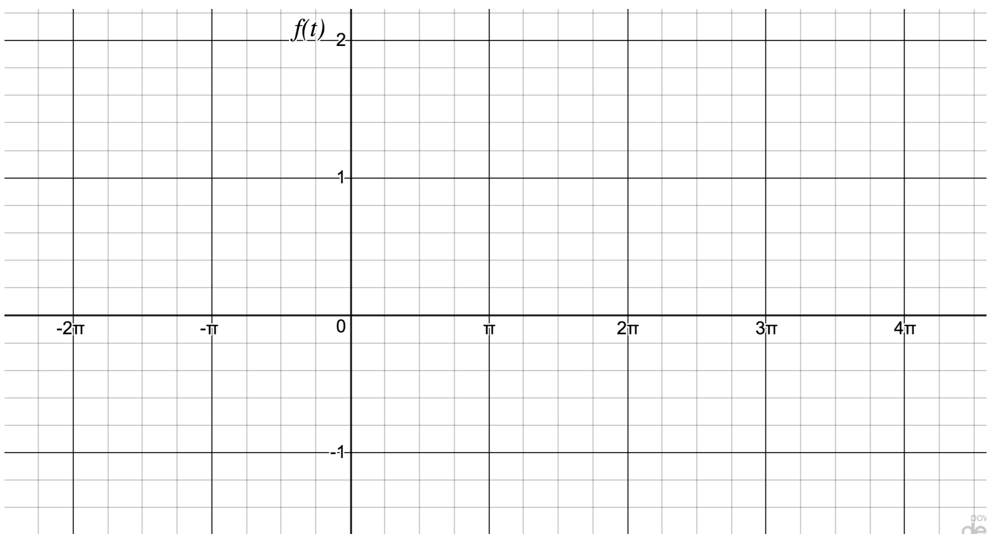
$0 \to \pi / 2$

$\pi / 2 \to \pi$ _________________________

$\pi \to 3\pi / 2$ _________________________

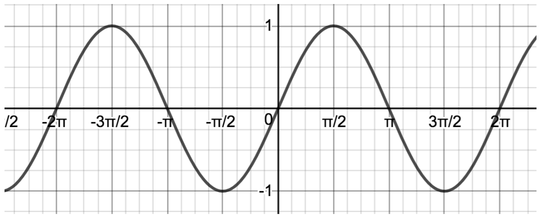
$3\pi / 2 \to 2\pi$ _________________________

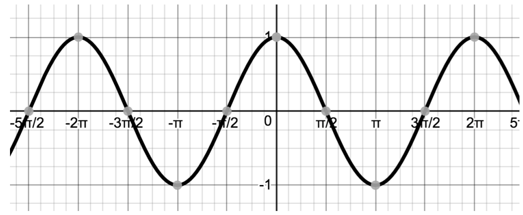
$2\pi \to 5\pi / 2$ _________________________



See the animation https://www.geogebra.org/m/cNEtsbvC again

Note: On all Trigonometric graphs, it is expected that you show scale clearly and label coordinates of high points and low points on graph.





__________________________ ______________________________

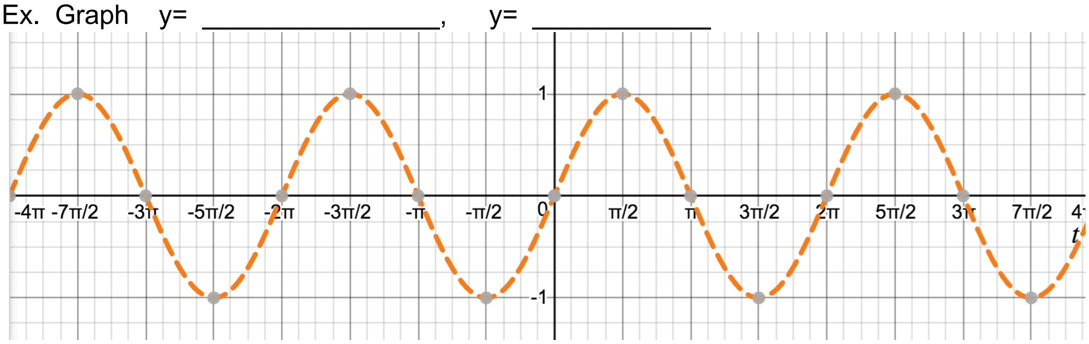
Both these graphs are ___________________________ with period _____________________ and have key points occuring every quadrantal angle or every __________________________

Transformations of the sine and cosine graphs.

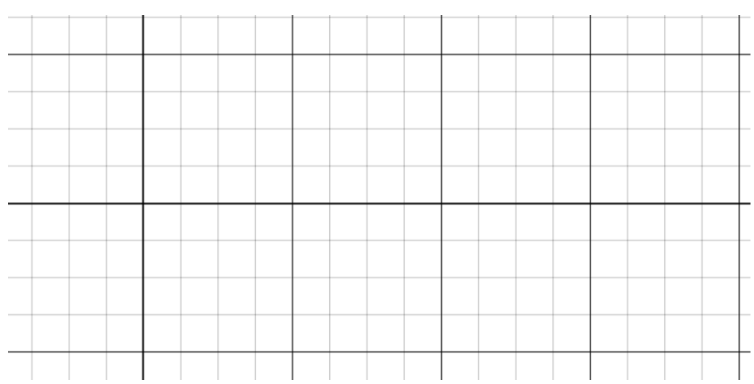
These two graphs can be used as basic graphs together with transformations (review 2.6 as needed).

Vertical Shift	$f(x)+c$ $f(x)-c$

Ex. Graph y= ___________________, y= ______________

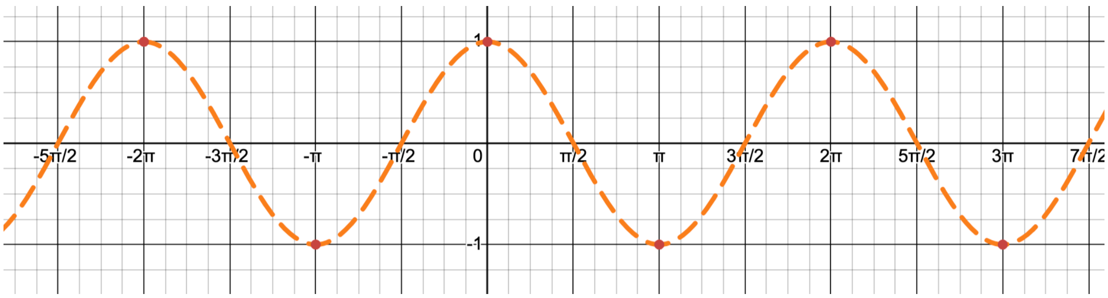


Ideally, eventually, rather than graph the original and then transform it, you would be able picture the transformation in your head to get a starting point, and then use the “quarter period pattern” to generate the rest.

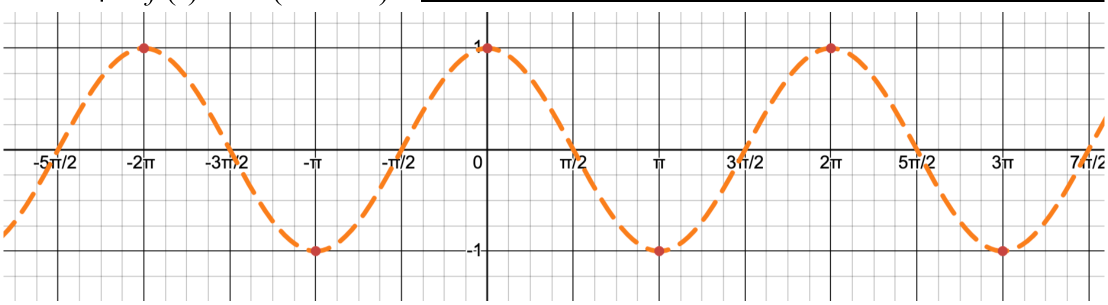


Horizontal Shift	$f(x+c)$ $f(x-c)$

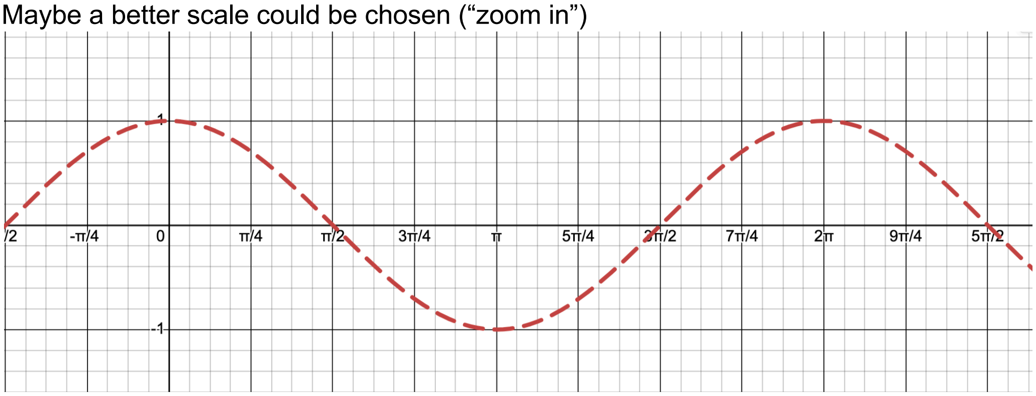
Ex. Graph $f(t)=\cos(t-\pi/4)$ ______________________________



Ex. Graph $f(t)=\cos(t+\pi/8)$ ______________________________

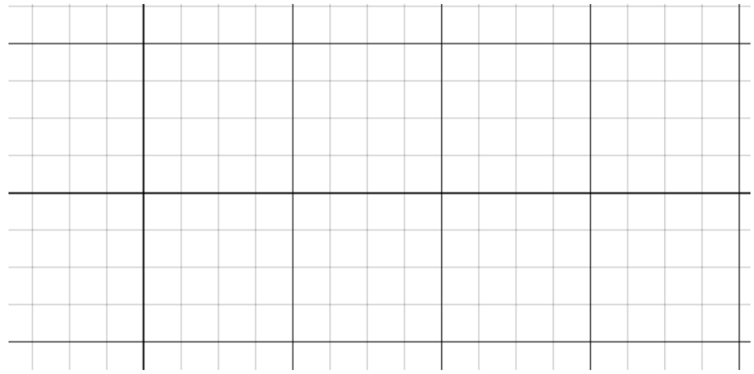


Maybe a better scale could be chosen ("zoom in")



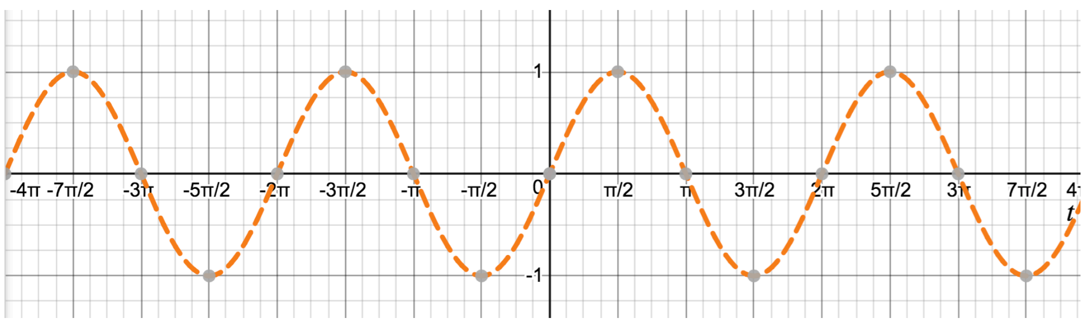
Graph $f(t) = \sin(t - \pi)$

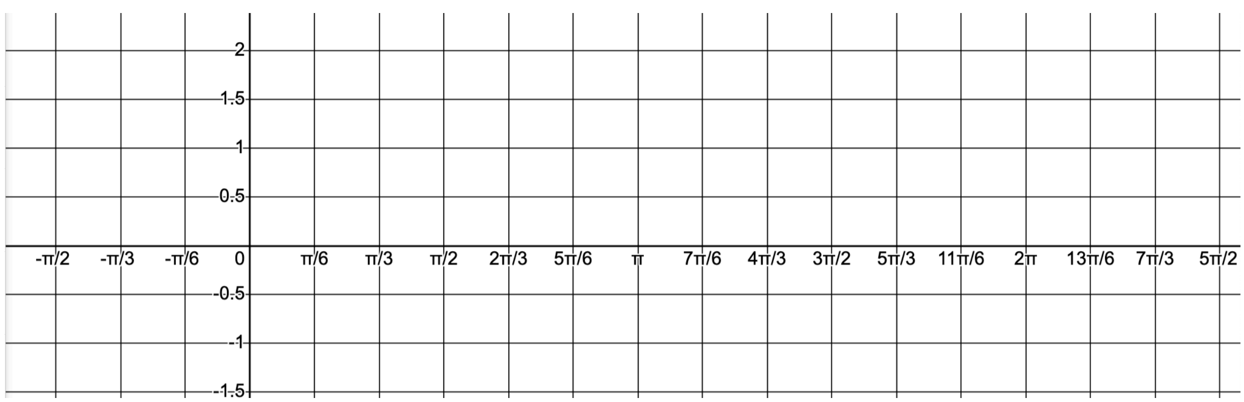
Again, rather than graph the original and then transform it, picture the transformation in your head to get a starting point, and then use the “quarter period pattern” to generate the rest



When graphing a sine or cosine graph, a choice of scale showing multiples of $\pi / 2$ is usually a good choice, but in some cases, a better choice can be made.

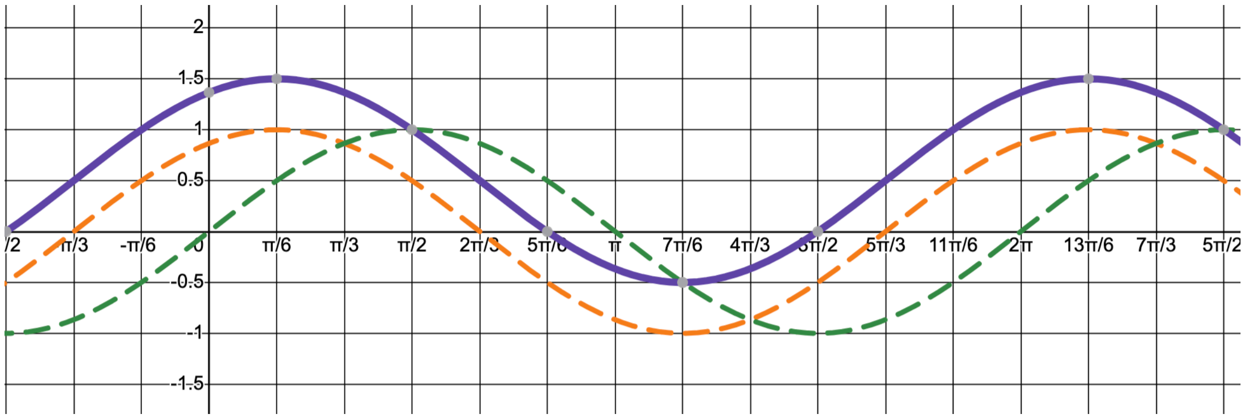
Graph $y = \sin\left(t + \dfrac{\pi}{3}\right)$ ____________________________



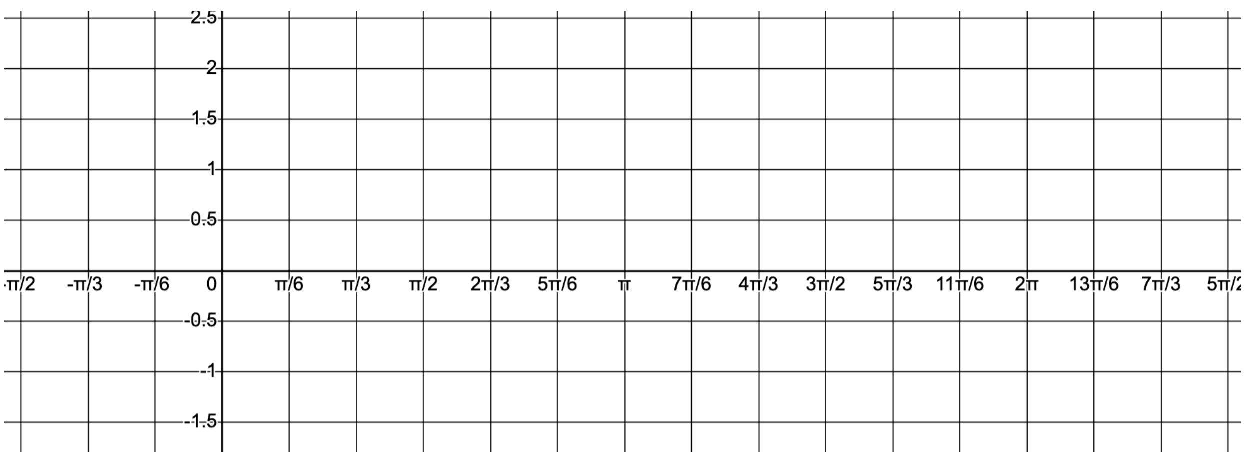


Combining Translations:

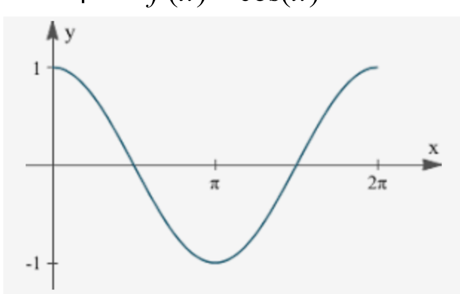
How would we graph $y = \sin\left(t + \frac{\pi}{3}\right) + \frac{1}{2}$?________________________________



And ideally, without having to sketch the intermediate stages.

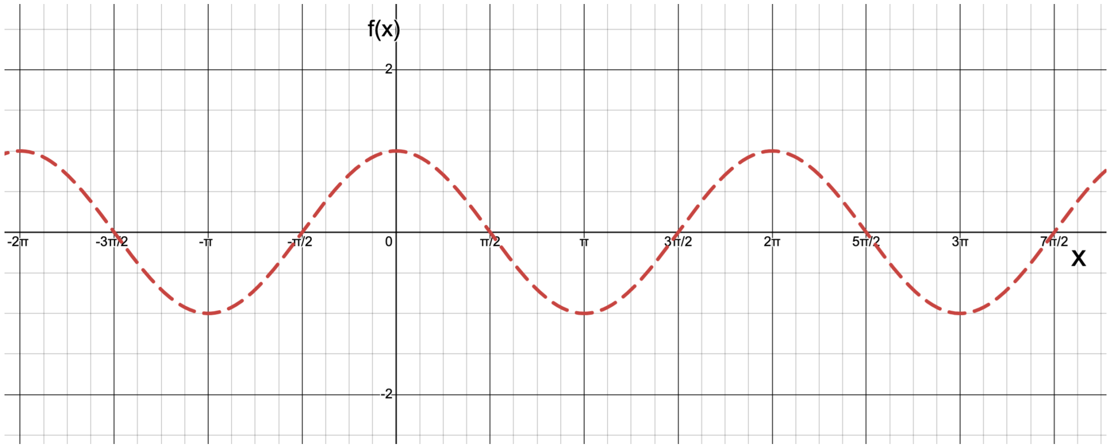


Note: At this point, as a convention, we switch our input from t to x but keep in mind, this x is not the same as the x value of the point on the unit circle.
For example: $f(x) = \cos(x)$

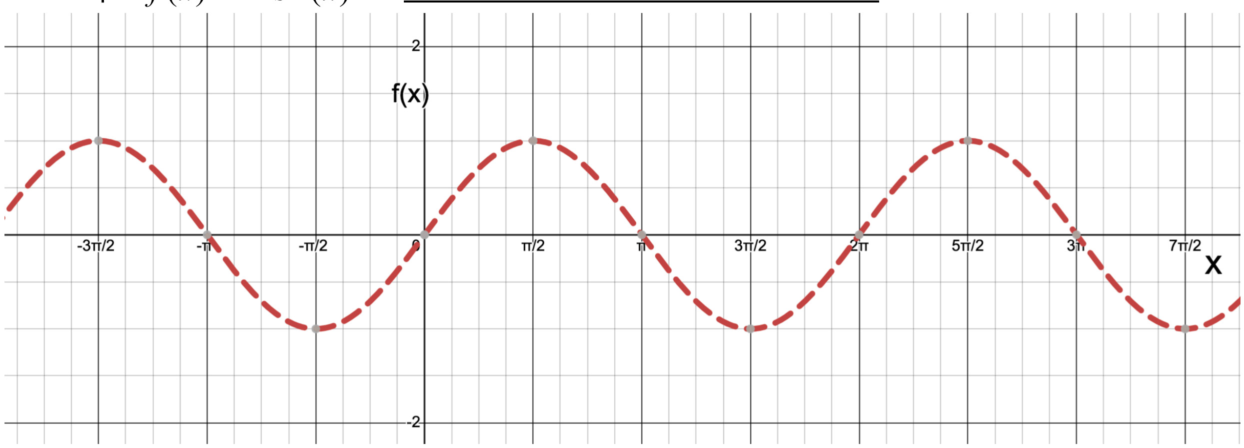


Vertical Stretch/Compress and Reflection. $y = c\,f(x)$

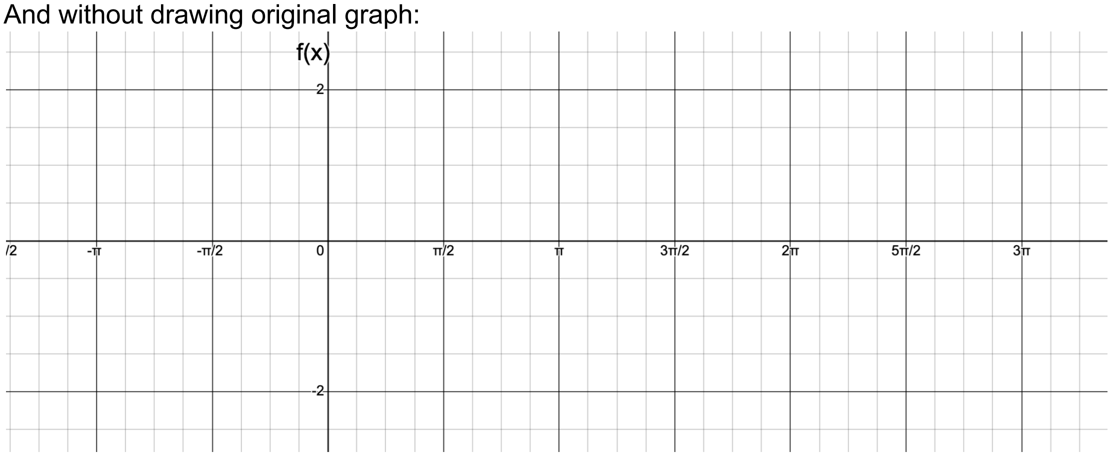
Ex. Graph $f(x) = 2\cos(x)$ $f(x) = \frac{1}{2}\cos(x)$



Ex. Graph $f(x) = -2\sin(x)$ ______________________



And without drawing original graph:

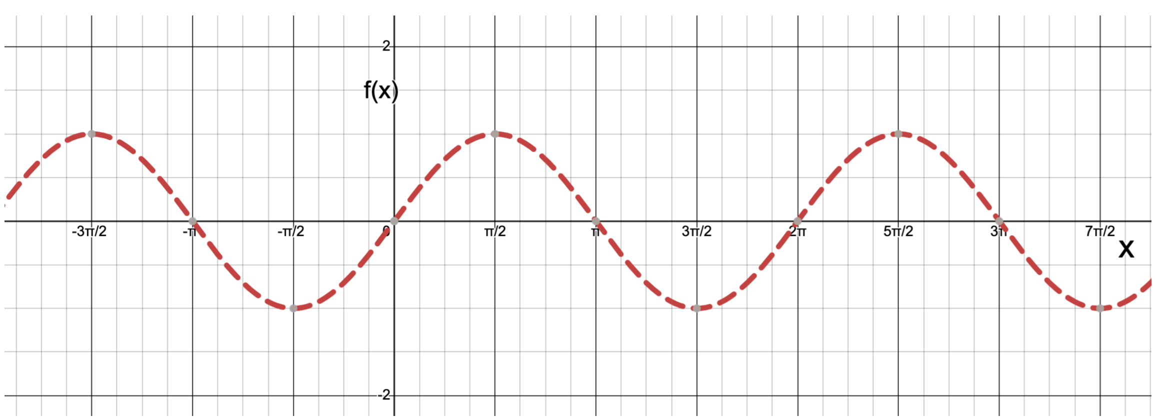


In general, for graphs of the form: $y = a\cos(t)$, $y = a\sin(t)$

5.3ii Graphing the Sine and Cosine Function part ii (period change)

Horizontal Stretch/Compress and Reflection. $y = f(cx)$

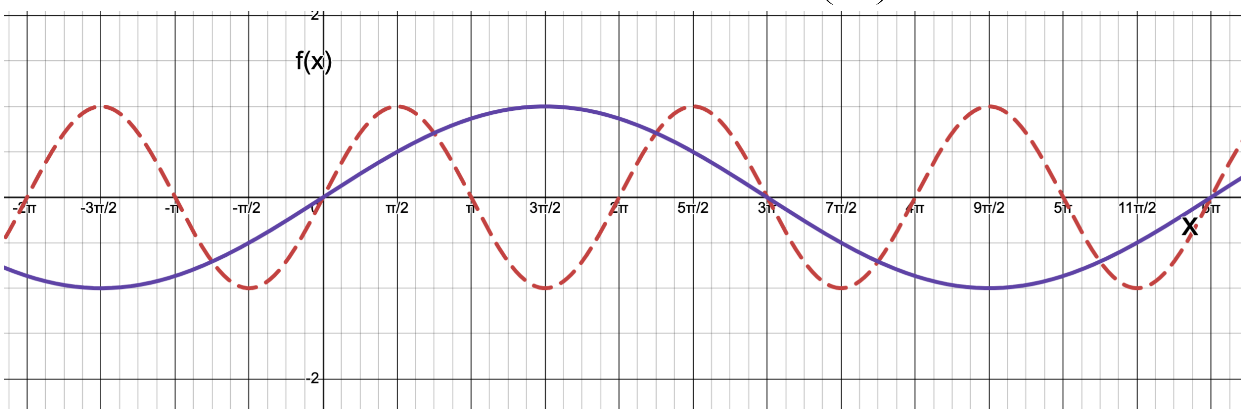
Graph $y = \sin(2x)$ ____



Initially, we might graph this by using our knowledge of horizontal compression or we night simply plot points (note: plotting points is inefficient and should be our last resource.)

Period?__________________________

Thus, the above graph is a horizontal compression whereas $y = \sin\left(\frac{1}{3}x\right)$ is a horizontal stretch.



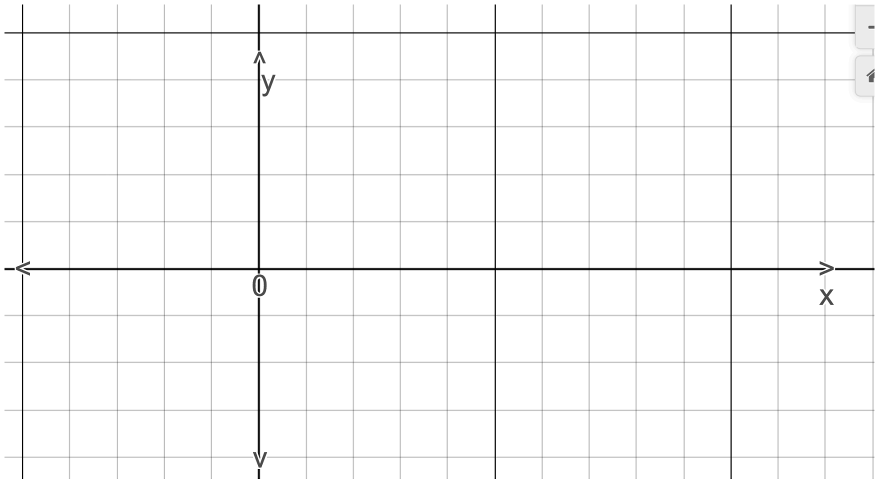
Period _________________?

In general, for graphs of the form: $y = \cos(kx)$, $y = \sin(kx)$ k has the effect of changing the ________________ to ____________________

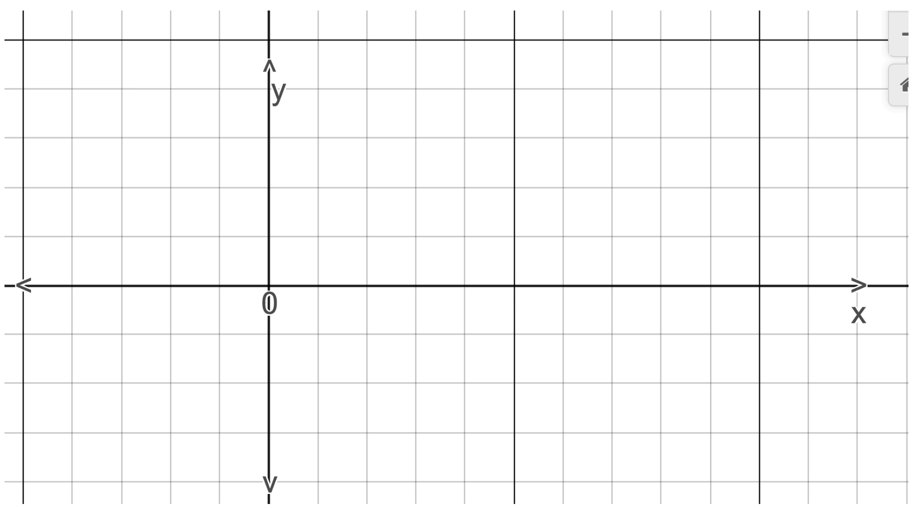
For this type of graph, rather than sketch the original graph and then stretch/compress it, we plan ahead and find the period. Then we break this period into fourths since the key points (lo-zero-hi-zero) occur every one-fourth of the period, and choose our x axis scale accordingly.

Reminder: On all Trigonometric graphs, it is expected that you show scale clearly and label coordinates of high points and low points on graph

Ex: Graph at least one period of________________________ .



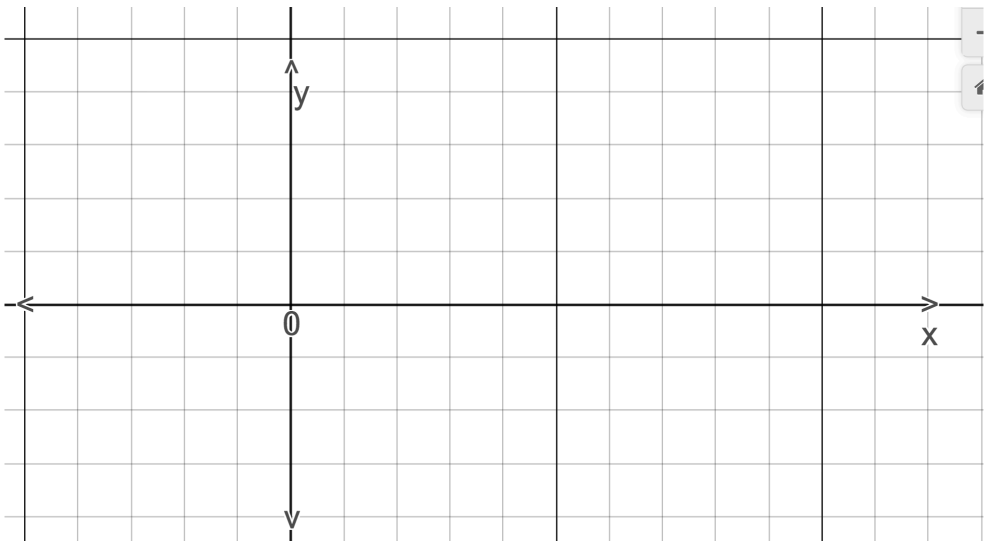
Ex: Graph at least one period of ____________________ .



Combining vertical and horizontal stretch/compress.

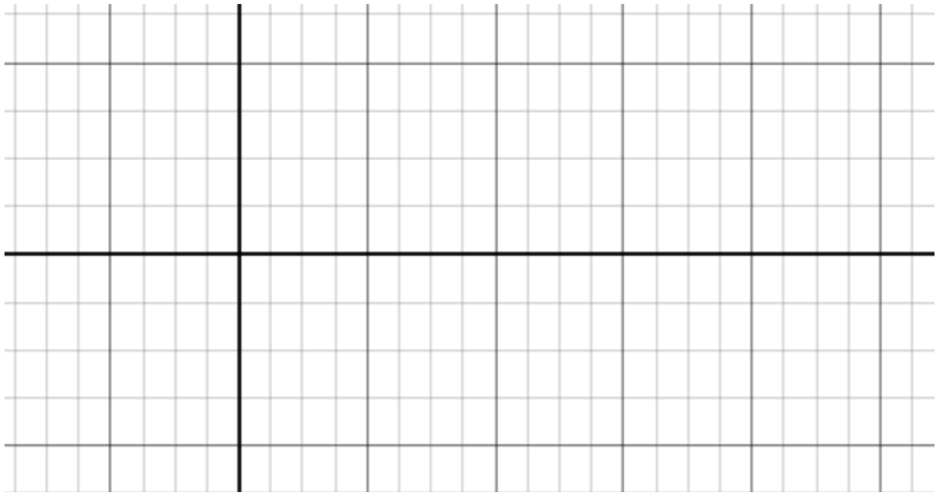
$y = a\cos(kx)$ $y = a\sin(kx)$

Ex:

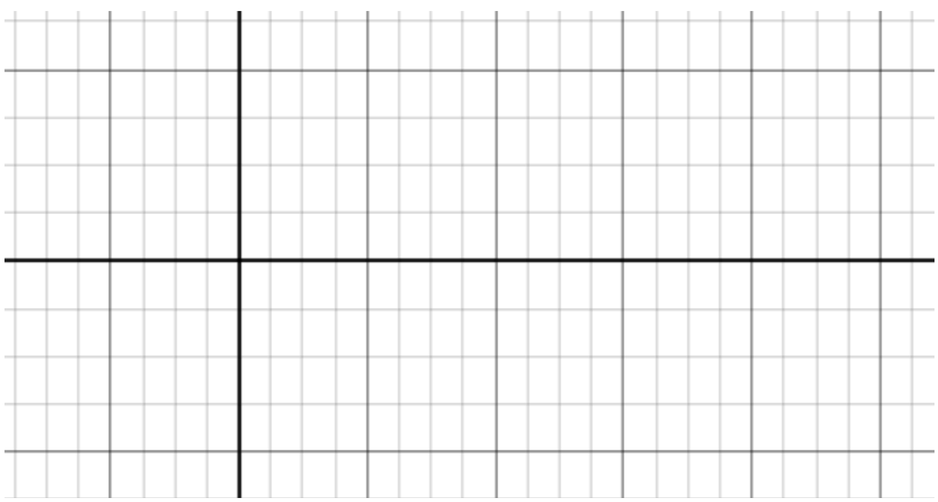


This next example will lead us into the third part of graphing sine and cosine where we put it all together.

Graph $f(x) = 4\sin(2x)$

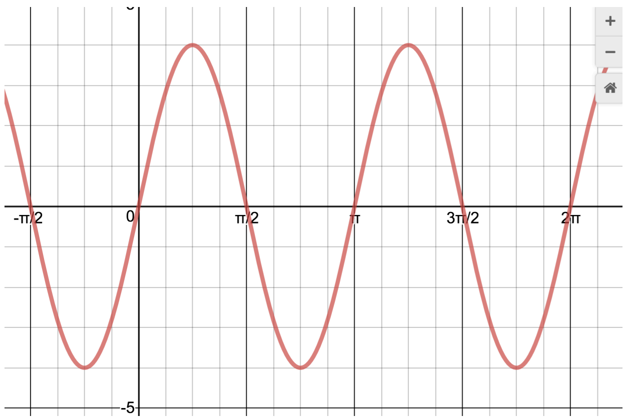


Use the above graph to graph $g(x) = 4\sin\left(2\left(x - \frac{\pi}{4}\right)\right)$



5.3iii Graphing the Sine and Cosine Function part iii : Putting it All Together

Use the graph of $f(x) = 4\sin(2x)$ to graph $g(x) = 4\sin\left(2\left(x - \frac{\pi}{4}\right)\right)$



Notice, the function $g(x)$ would normally be written ________________

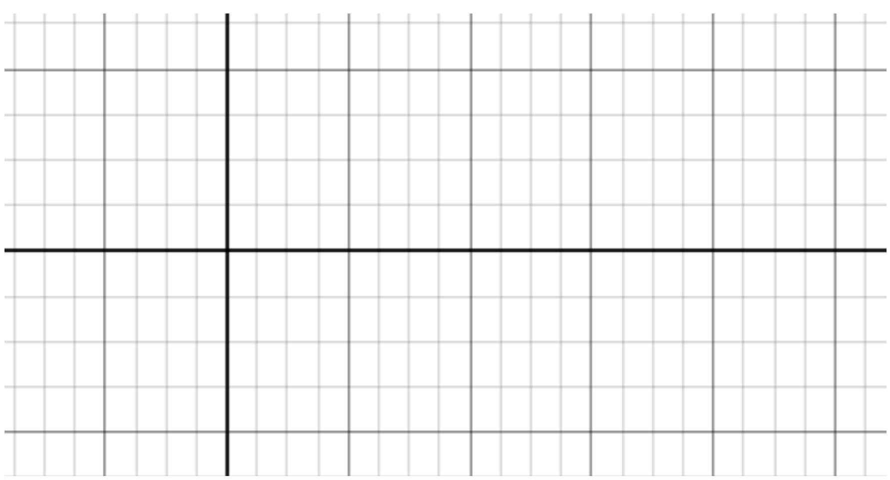
But what was the horizontal shift?__________________________

Note: The horizontal shift is NOT _______________________

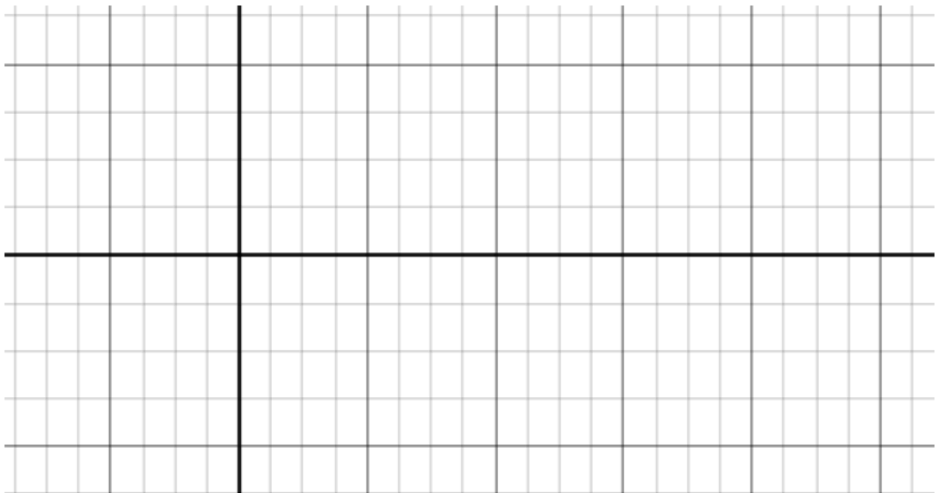
So given $g(x)$____________________ to find the horizontal shift, we have to factor out the 2.

Examples

Graph $f(x) = 2\sin\left(3x - \frac{\pi}{4}\right)$



Graph $f(x) = 3\cos\left(\pi x + \frac{\pi}{6}\right)$



How would we graph $g(x) = -3\cos\left(\pi x + \frac{\pi}{6}\right) + 1$?____________________________________

Summarizing $f(x) = a\cos\big(k(x+b)\big) + c$ $\qquad$ $f(x) = a\sin\big(k(x+b)\big) + c$

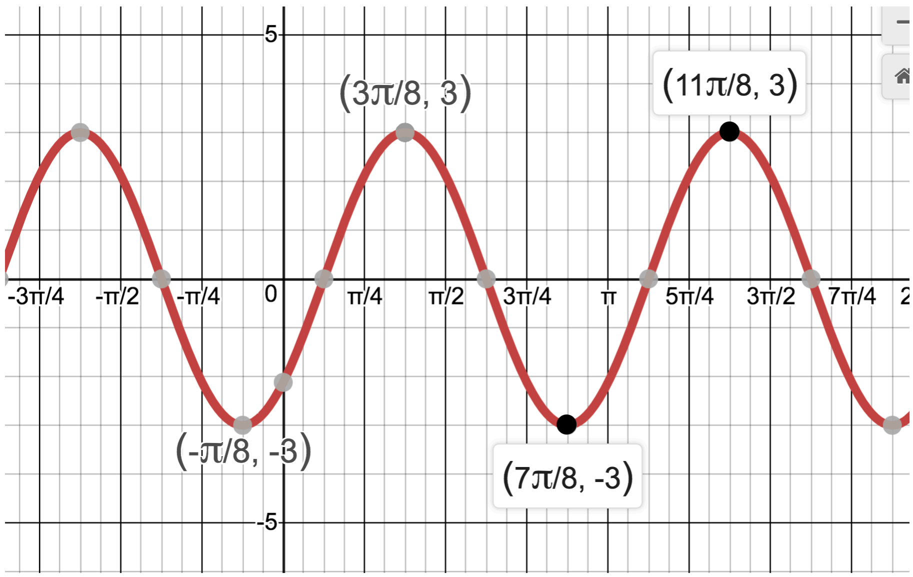
Using a graph to find the equation.

Often, we are provided with observational data and we wish to find an equation of the form

$$f(x) = a\cos\big(k(x+b)\big)+c \qquad f(x) = a\sin\big(k(x+b)\big)+c$$

to model the physical situation.

Find an equation corresponding the graph below. For one of the labeled points, check that it satisfies your equation.



Measure the amplitude, half the distance from the lowest point to the highest. This is a .

Measure the period. Use this to get κ since 2π/κ is the period.

Now to finish, we need to find the horizontal shift. Read the shift from the graph. There are many possible answers depending on whether you are picturing it as a shift of the cosine graph or the sine graph. Put the shift into the factored form of the equation.

Check one of the given points in your equation.

5.6. Simple Harmonic Motion *(5.5 will be covered next unit. 5.4 will be covered shortly)*

Periodic behavior is very common in nature. Here, we look at an introduction to Simple Harmonic Motion.

If the displacement, y, of an object at time t can be expressed by

$y = a\cos(\omega t)$ or $y = a\sin(\omega t)$

then the object is said to be in simple harmonic motion where

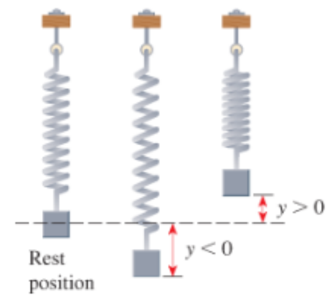
Amplitude = _________

Period = ____________

and Frequency = ____________

Modeling a Mass-Spring System:

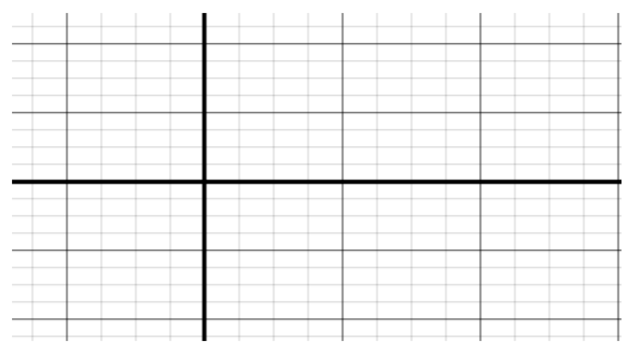
Suppose the displacement of a mass suspended by a spring is modeled by $y = 3\sin(2\pi t)$ *(where y is measured in cm. time in seconds)*



a) Find the amplitude, period and frequency of the motion.

b) Describe the motion

c) Graph displacement as a function of time.



d) How does the motion change if the system is modeled by

(i) $y = -3\sin(2\pi t)$

(i). $y = 3\cos(2\pi t)$

(iii). $y = -3\cos(2\pi t)$

See other example in book including damped harmonic motion, and phase shift

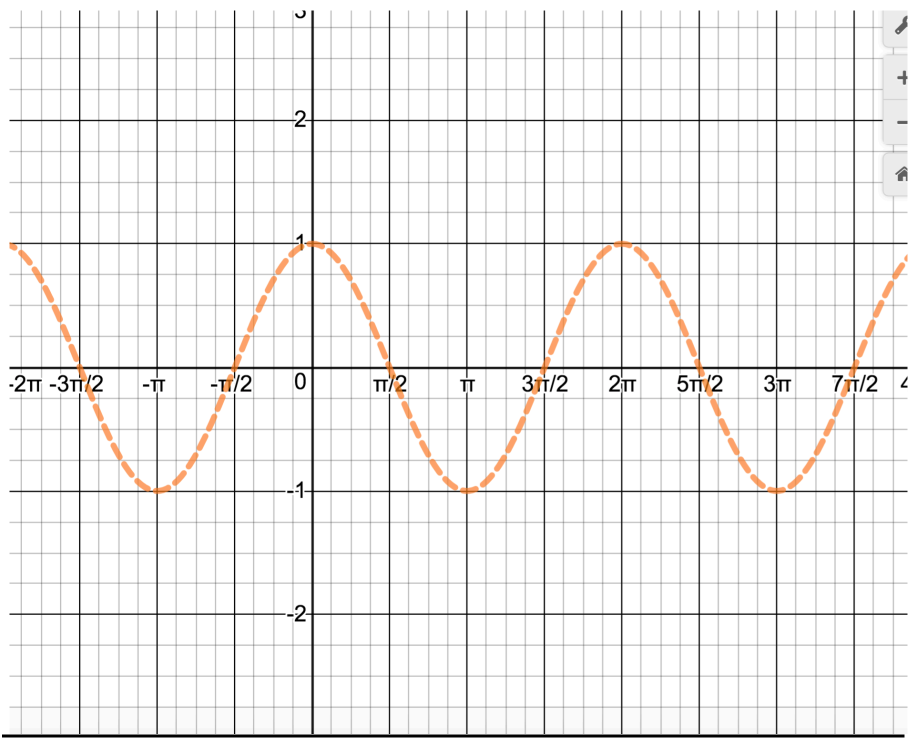
5.4: Graphs of Other Trigonometric Functions.

For the graphs of secant and cosecant, we can use our knowledge of graphing cosine and sine, together with the reciprocal relationships:

Ex: $f(x)=\sec(x)$

Since $f(x)=\sec(x)=\dfrac{1}{\cos(x)}$, everywhere that ________________, $f(x)=\sec(x)$ is ____________, thus where the graph of $g(x)=\cos(x)$ has an __________________________ the graph of $f(x)=\sec(x)$ has a ____________________.

Also, as $g(x)=\left|\cos(x)\right|$ gets small $f(x)=\left|\sec(x)\right|$ _____________

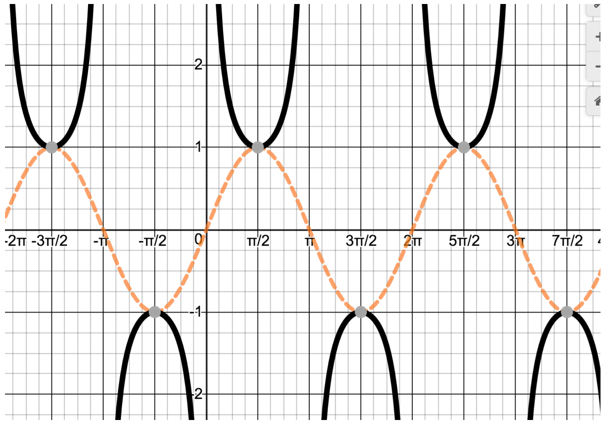


Previously, by the unit circle definition, we determined the domain of $f(x)=\sec(x)$ to be

_____________________________ which we can now see on the graph. In addition, the graph tells us that the range of $f(x)=\sec(x)$ is ________________________________

What is the period of $f(x)=\sec(x)$? ________________________________

The graph of $f(x)=\csc(x)$ can be found similarly (see text).



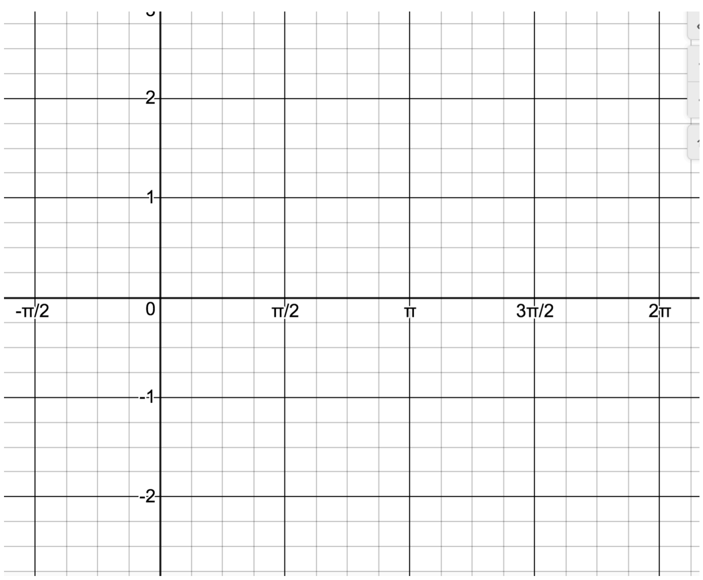
Notice the location of the vertical asymptotes. How would we find them algebraically?
Note domain, range, period

How would you graph $f(x)=\csc\left(x-\frac{\pi}{4}\right)$ l?

Note: Even though we could use the above graph together with a

transformation (______________________________), it is actually easier to

____

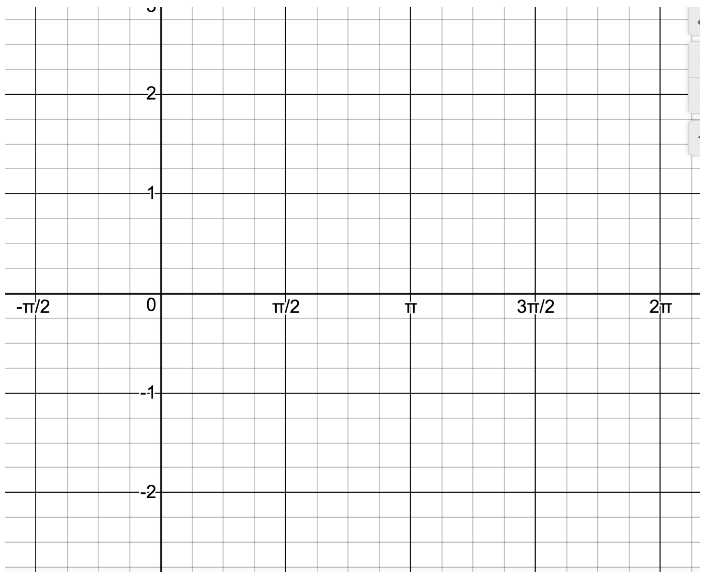


EX: Sketch the graph of $f(x) = 3\csc(2x)$.

Note: Even though we could use the above graph together with a

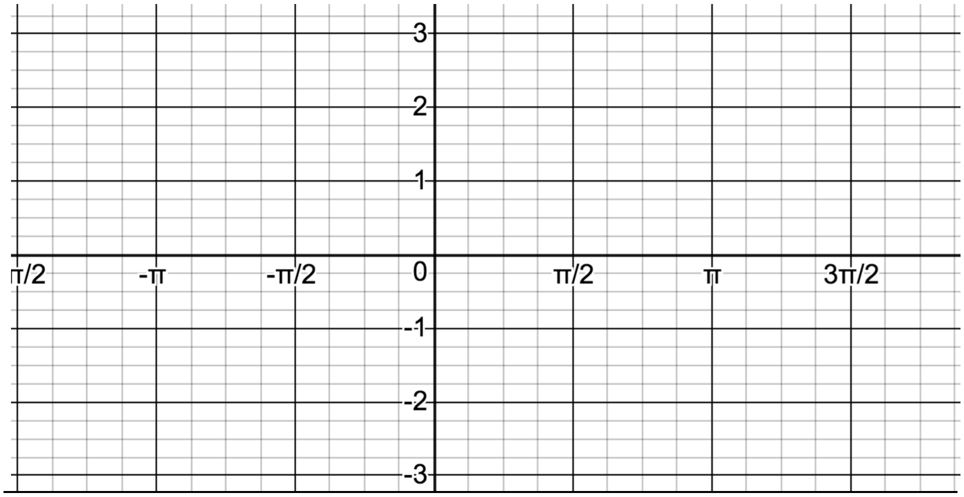
transformation (______________________________), it is actually easier to

____



How would we find the domain and asymptotes algebraically if we didn't have the graph?

The graph of $f(x) = \tan(x)$



Discuss domain, range, period, odd, asymptotes…

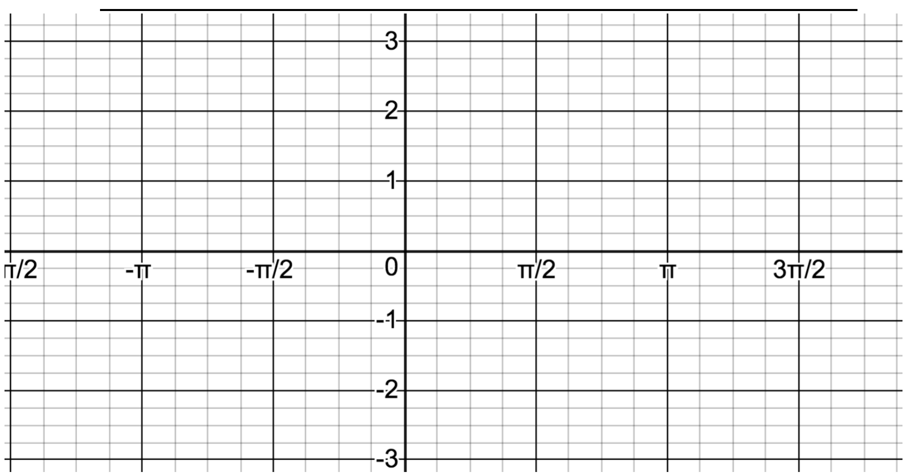
EX: Sketch the graph of $f(x) = \tan(2x)$

.

Note: Even though we could use the above graph together with a

transformation (______________________________), it is actually easier to

____

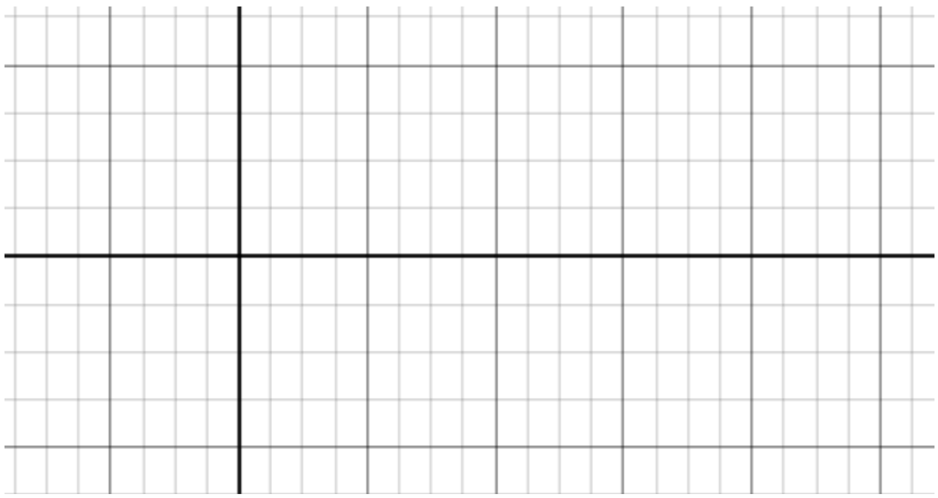


What is the period? ________________________________

In general $f(x) = a\tan(kx)$ has period____________________ Find asymptotes by considering where the denominator $\cos(kx) = 0$ Midway between asymptotes is an x intercept, midway between the x intercept and an asymptote, $f(x) = a$ or $f(x) = -a$ (Note: a is not called the "amplitude" here)

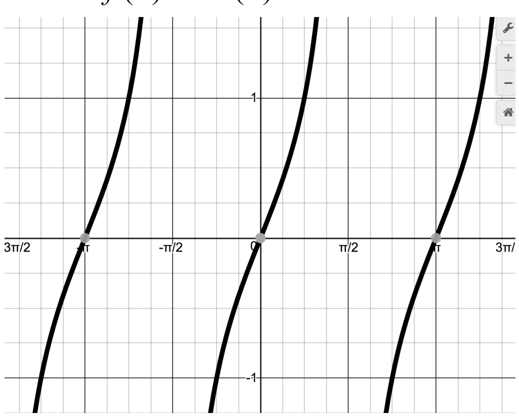
EX: Sketch the graph of $f(x) = 2\tan\left(\dfrac{\pi}{3}x\right)$

.

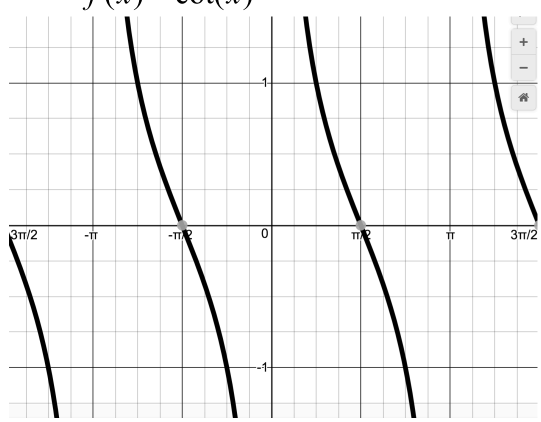


Graph of $f(x) = \cot(x)$

$f(x) = \tan(x)$



$f(x) = \cot(x)$



Discuss domain, range, period, odd, asymptotes…

Note: the asymptotes for tangent and cotangent are not in the same location (why?). In addition notice that the tangent graph increases between each pair of asymptotes where the cotangent decreases.
See text for more examples